Problem 1

University Physics, problem 51
A point charge $q_1 = 4.0$ nC is placed at the origin, and a second point charge $q_2 = -3.00$ nC is placed on the x-axis at $x = +20.0$ cm. A third point charge $q_3 = 2.00$ nC is to be placed on the x-axis between and (Take as zero the potential energy of the three charges when they are infinitely far apart).

a. What is the potential energy of the system of the three charges if q_3 is placed at $x = +10.0$ cm.

b. Where should q_3 be placed to make the potential energy of the system equal to zero?

Solution

Part (a)

As we know, the electric potential energy, just like gravitational, is a shared property of the two bodies. We cannot say the potential energy of q_1 or the potential energy of the earth, it's always the potential energy of a system of two bodies. Here, we have three smaller systems, q_1 and q_2, q_2 and q_3 and q_1 and q_3. Each system has its own potential energy, and the three potential energies add up to form the potential energy of the bigger system.

$$U_T = U_{12} + U_{23} + U_{13}$$
$$U_T = K_e\frac{q_1q_2}{r_{12}^2} + K_e\frac{q_2q_3}{r_{23}^2} + K_e\frac{q_1q_3}{r_{13}^2}$$
$$U_T = 9\times10^9\left(\frac{4\times10^{-9}\cdot -3\times10^{-9}}{20\times10^{-2}} + \frac{-3\times10^{-9}\cdot 2\times10^{-9}}{10\times10^{-2}} + \frac{4\times10^{-9}\cdot 2\times10^{-9}}{10\times10^{-2}}\right) = \boxed{-3.6\times10^{-7}\text{J}}$$

Part (b)

Let the charge q_3 be placed at an arbitrary point x along the x-axis. Note that the problem states that q_3 lies between q_1 and q_2, thus $0 < x < 20cm$.

$$U_T = U_{12} + U_{23} + U_{13}$$
$$U_T = K_e\frac{q_1q_2}{r_{12}^2} + K_e\frac{q_2q_3}{r_{23}^2} + K_e\frac{q_1q_3}{r_{13}^2}$$
$$0 = 9\times10^9\left(\frac{4\times10^{-9}\cdot -3\times10^{-9}}{20\times10^{-2}} + \frac{-3\times10^{-9}\cdot 2\times10^{-9}}{(20-x)\times10^{-2}} + \frac{4\times10^{-9}\cdot 2\times10^{-9}}{x\times10^{-2}}\right)$$
$$\frac{-12}{20} + \frac{-6}{20-x} + \frac{8}{x} = 0 \implies \frac{160-14x}{20x-x^2} = \frac{3}{5} \implies 3x^2 - 130x + 800 = 0$$

Solving this quadratic equation gives the two solutions $x = 35.9$ cm and $x = 7.42$ cm. The first solution is refused as it is greater than 20.0cm, thus the solution to the problem is $\boxed{x = 7.42 \text{ cm}}$

Problem 2

University Physics, problem 84
Using the electric field expressions below:

$$E = (K_e Q)(\frac{r}{R^3})(4 - \frac{3r}{R}) \qquad (r \leq R)$$

$$E = K_e \frac{Q}{r^2} \qquad (r \geq R)$$

a. Show that for $r \geq R$ the potential is identical to that produced by a point charge (Take the potential to be zero at infinity.)

b. Obtain an expression for the electric potential valid in the region $r \leq R$.

Note: The textbook asks you to use the solutions to problem 84A. I directly included the solutions as if they were given, but I highly encourage you to solve problem 84A as it has some important concepts.

Part (a)

The problem specifies that the potential at infinity is zero.

$$V(r) - V(\infty) = -\int_{\infty}^{r} K\frac{Q}{r^2}dr = -\lim_{t\to\infty}\int_{t}^{r} K\frac{Q}{r^2}dr = \lim_{t\to\infty}(K\frac{Q}{r} - K\frac{Q}{t})$$

$$V(r) = K\frac{Q}{r}$$

It's very important to check that the integral bounds match the bounds of the electric field expression you are integrating. In this case, it's always greater than R so it checks out. The expression we obained for $V(r)$ matches that of a point charge. If you are having trouble with improper integrals, check your MATH-152 lecture notes or textbook.

Part (b)

$$V(r) - V(\infty) = -\int_{\infty}^{r}(K_e Q)(\frac{r}{R^3})(4 - \frac{3r}{R})dr$$

WRONG!!

The electric field expression used in the integral is valid for the region where $r \leq R$, while the integral extends to the region beyond that and "up to" infinity. Even if you try to solve the improper integral, you will find that it is divergent, so that will give you a hint that you're doing something wrong. There are two ways to circumvent this, the first is to split up the integral into two.

$$V(r) - V(\infty) = -\int_{\infty}^{R} K\frac{Q}{r^2}dr - \int_{R}^{r}(K_e Q)(\frac{r}{R^3})(4 - \frac{3r}{R})dr = K\frac{Q}{R} - \int_{R}^{r}(K_e Q)(\frac{r}{R^3})(4 - \frac{3r}{R})dr$$

The first integral is merely the expression found in part a evaluated at $r = R$. Solving the second integral is a matter of long algebra. Solving and simplifying will net you with this expression:

$$\boxed{V(r) = \frac{KQ}{R}\left(\frac{r^3}{R^3} - \frac{r^2}{R^2} + 2\right)}$$

The other way to solve this is to just solve for $V(r) - V(R)$ and redefine the integral bounds accordingly. V(R) here will obviously not be zero (hint: it's equal to the expression found in part (a) evaluated at $r = R$).