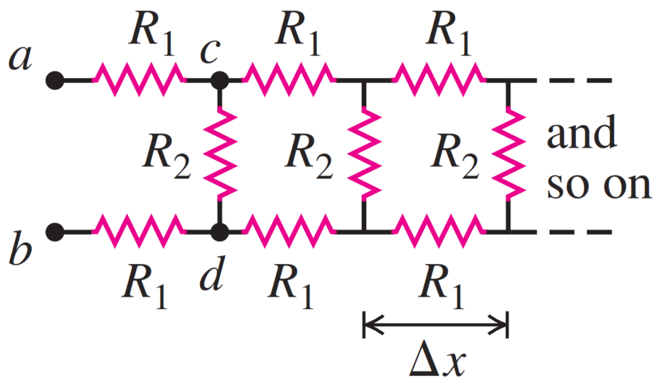
Problem 1

University Physics, chapter 6, problem 91
As shown in the figure, a network of resistors of resistances R_1 and R_2 extends to infinity toward the right. Prove that the total resistance of the infinite network is
$R_T = R_1 + \sqrt{R_1^2 + 2R_1R_2}$.



Solution

Since the network extends to infinity, then we can consider that all the network after CD is equivalent to R_T. This means that we have a network with R_1 in series with an R_2 and R_T parallel combination, which in turn is in series with another R_1.

$$\frac{1}{R_{T,2}} = \frac{1}{R_2} + \frac{1}{R_T} \implies R_{T,2} = \frac{R_T R_2}{R_T + R_2}$$

$$R_T = 2R_1 + R_{T,2} = 2R_1 + \frac{R_T R_2}{R_T + R_2} \implies R_2R_T + R_T^2 = 2R_1R_2 + 2R_1R_T + R_2R_T$$

$$R_T^2 - 2R_1R_T - 2R_1R_2 = 0 \implies R_T = \frac{2R_1 \pm \sqrt{4R_1^2 + 8R_1R_2}}{2}$$

$$R_T = R_1 \pm \sqrt{R_1^2 + 2R_1R_2}$$

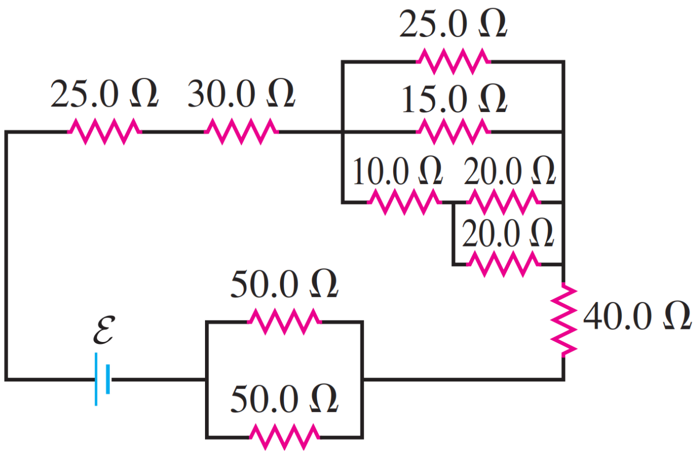
The negative solution is refused since resistance cannot be negative

$$\boxed{R_T = R_1 + \sqrt{R_1^2 + 2R_1R_2}}$$

Problem 2

University Physics, chapter 6, problem 70

In the circuit shown in the figure, all the resistors are rated at a maximum power of 2.00 W. What is the maximum emf $\mathcal{E}$ that the battery can have without burning up any of the resistors?



Solution

The power rating of a resistor means the maximum it will function correctly at. We know that the power dissipated in a resistor is $P_R = I^2R$. Since R is constant for a given resistor and we already know the maximum power, we need to find the resistor with the maximum resistance in the circuit (since resistance will be inversely proportional to the square of the current through it), then this current can be used to solve for $\mathcal{E}$.

We start by simplifying the upper right corner. 20Ω and 20Ω in parallel gives us 10Ω, which is in series with 10Ω, giving us 20Ω. 20Ω in parallel with 15Ω and 25Ω gives us $\frac{300}{47} \approx 6.38\Omega$. The lower middle combination of 2 50Ω in parallel simplifies to 25Ω. Since the current in the equivalent resistor of any parallel combination is the sum of that through the individual resistors, then we know for sure that the current through this 25Ω and 6.38Ω is greater than that of their own network's constituents.

We thus end up with the following resistors in series: 25Ω, 30Ω, 6.38Ω, 40Ω and 25Ω. The 40Ω resistor has the maximum resistance and can thus it is the resistor that needs to have the maximum current possible.

Example: if, for example, we find the current as $0.28A$. The power rating of the 25Ω resistor will be 2 W while that of the 40Ω resistor will be 3.2 W, which is more than the power rating. Thus the maximum possible current in the circuit must be solved for using the 40Ω resistor.

$$P = I^2R \implies I = \sqrt{P/R} = \sqrt{2/40} = \frac{\sqrt{5}}{10}$$

$$R_{eq} = 25 + 30 + 6.38 + 40 + 25 = 126.38\Omega$$

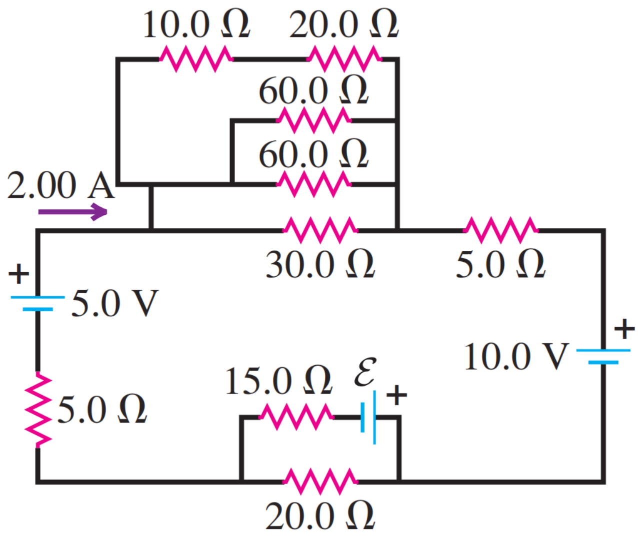
$$\mathcal{E} = IR = 126.38 \cdot \frac{\sqrt{5}}{10} = \boxed{28.2 \text{ V}}$$

Problem 3

University Physics, chapter 6, problem 68
Consider the circuit shown in the figure.

a. What must the emf $\mathcal{E}$ of the battery be in order for a current of 2.00 A to flow through the 5.00-V battery as shown? Is the polarity of the battery correct as shown?

b. How long does it take for 60.0 J of thermal energy to be produced in the 10Ω resistor?



Solution

Solution

Part (a)

As always, we start by simplifying the circuit. As for the parallel combination at the top, 60Ω and 60Ω in parallel gives 30Ω. 10Ω and 20Ω in series also gives 30Ω 3x 30Ω in parallel gives 10Ω. Now we take 2 loops. First loop is the biggest loop, clockwise direction, travels through the bottom 20Ω resistor. Loop 2 is counterclockwise, small loop which contains unknown emf. Let the current through the 20Ω resistor be I and thus the current through the 15Ω resistor is $2 - I$.

$-5 + 2 \cdot 10 + 2 \cdot 5 + 10 + 2 \cdot I + 2 \cdot 5 = 0$

$I = -2.25A$, the negative means that it is opposite to the direction we set at first, i.e. counterclockwise.

$-\mathcal{E} + 20 \cdot -2.25 - 15 \cdot (2 - (-2.25)) = 0 \implies \boxed{\mathcal{E} = -108.75\,\text{V}}$

$\mathcal{E}$ is negative thus polarity of battery is incorrect and it should be flipped.

Part (b)

The closest thing we have to energy is power, where $P = I^2R$, thus we need to know the current through the 10Ω resistor. The voltage across the equivalent resistor of the top network is the same as that across the 10Ω and 20Ω series combination.

$V_{eq} = IR_{eq} = 2 \cdot 10 = 20\text{V}$
$I_{10\Omega} = I_{20\Omega} = 20/30 = 2/3\,\text{A}$
$P = (2/3)^2 \cdot 10 = 40/9\,\text{W}$
$P = E/t \implies t = E/P = 60 \cdot 9/40 = \boxed{13.5 \text{ s}}$