Problem 1

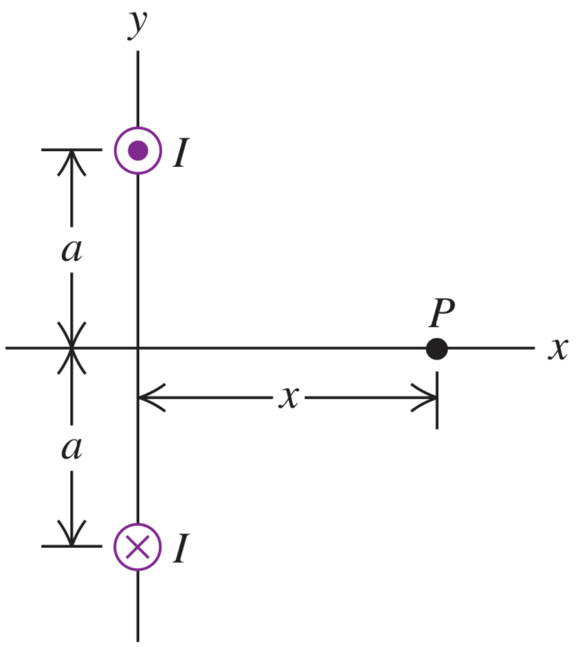
University Physics, Problem 68

The figure shows an end view of two long, parallel wires perpendicular to the xy-plane, each carrying a current I but in opposite directions.

a. Derive the expression for the magnitude of at any point on the x-axis in terms of the x-coordinate of the point. What is the direction of $\vec{\mathbf{B}}$?

b. At what value of x is the magnitude of a maximum?

c. What is the magnitude of $\vec{\mathbf{B}}$ when $x >> a$



Solution

Part (a)

The y-components of the two fields cancel out. The x-components add up.

$$B = 2B_x = 2B_1 \sin\theta = 2\frac{\mu_0 I}{2\pi\sqrt{x^2+a^2}} \cdot \frac{a}{\sqrt{x^2+a^2}} = \frac{\mu_0 I a}{\pi(x^2+a^2)}$$

Part (b)

Field maximum when x = 0 (inverse relation between B and x^2).

The way to do this rigorously is by taking the first derivative and equation it to zero, which gives $x = 0$ and $x = \infty$ as critical points. You need to check which is a maximum and which is a minimum, and the simplest way to do it is to plug in the values and compare.

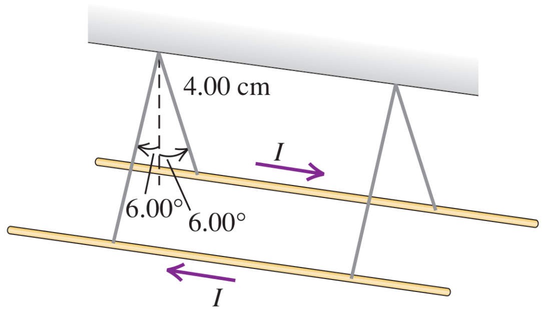
Part (c)

When $x >> a$, $B = \frac{\mu_0 I a}{\pi x^2}$, as $\sqrt{x^2+a^2} \approx \sqrt{x^2} = x$

Problem 2

University Physics, Problem 68

Two long, parallel wires hang by 4.00-cm-long cords from a common axis. The wires have a mass per unit length 0.0125 kg/m and carry the same current in opposite directions. What is the current in each wire if the cords hang at an angle of 6^o with the vertical?



Solution

The wires are in static equilibrium. Taking the forces on one wire:

$$T\cos\theta = mg \tag{1}$$

$$T\sin\theta = F_B = BIL = \frac{\mu_0 I}{2\pi r} IL = \frac{\mu_0 I^2 L}{2\pi r} \tag{2}$$

Dividing 2 by 1:

$$\tan\theta = \frac{\mu_0 I^2 L}{2\pi r m g} = \frac{\mu_0 I^2}{2\pi r g \lambda_m}$$

$$I^2 = \frac{2\pi r g \lambda_m \tan\theta}{\mu_0}$$

$$r = 0.04\sin 6^o$$

$$I = \sqrt{\frac{2\pi(0.04\sin 6^o)(9.8)(0.0125)(\tan 6^o)}{4\pi\times 10^{-7}}} = \boxed{23.2\text{ A}}$$