Problem 1.
University Physics P. 337 (13th edition), Problem 58

A conducting rod with length $L = 0.200$ m, mass $m = 0.120$ kg , and resistance $R = 80.0\Omega$ moves without friction on metal rails as shown in the figure. A uniform magnetic eld with magnitude $B = 1.50$ T is directed into the plane of the gure. The rod is initially at rest, and then a constant force with magnitude $F = 1.90$ N and directed to the right is applied to the bar. How many seconds after the force is applied does the bar reach a speed of $v = 25.0$ m/s?

Given

$$
\begin{aligned}
L &= 0.200\,\mathrm{m} \\
m &= 0.120\,\mathrm{kg} \\
R &= 80.0\,\Omega \\
B &= 1.50\,\mathrm{T}, \text{ into the plane of the figure.} \\
F &= 1.90\,\mathrm{N}, \text{ directed to the right.} \\
v_0 &= 0.00\,\mathrm{m/s} \\
v &= 25.0\,\mathrm{m/s}
\end{aligned}
$$

Solution

When the rod moves to the right in the magnetic field, the current flows in a counterclockwise direction, according to Lenz's Law or the RHR or Faraday's Law. A wire carrying current moving in a magnetic field is subjected to a force $\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$, and according to the RHR, The direction of $\vec{\mathbf{F}}_B$ opposes $\vec{\mathbf{F}}$.
Therefore we have:

$$\Sigma\vec{\mathbf{F}} = \vec{\mathbf{F}} + \vec{\mathbf{F}}_B = m\vec{\mathbf{a}} \implies F - F_B = F - BIL = ma$$

$$\mathcal{E} = BLv \text{ and } I = \varepsilon/R \implies I = BLv/R$$

$$a = \frac{F - \frac{B^2L^2v}{R}}{m} = \frac{FR - B^2L^2v}{mR} \implies \frac{dv}{dt} = \frac{FR - B^2L^2v}{mR}$$

To solve this simple differential equation, we have to separate the variables then integrate.
To simplify the process, let's declare two constant α and β where $\alpha = FR$ and $\beta = B^2L^2$. The equation becomes:

$$\frac{dv}{dt} = \frac{\alpha - \beta v}{mR} \implies \frac{dv}{\alpha - \beta v} = \frac{dt}{mR}$$

Now that we have successfully separated the variables, it's time to integrate.

$$\int_0^v \frac{dv'}{\alpha - \beta v'} = \int_0^t \frac{dt'}{mR}$$

$$\frac{-1}{\beta}(\ln|\alpha - \beta v'|)\bigg|_0^v = \frac{t}{mR} \implies \frac{1}{\beta}(\ln|\alpha| - \ln|\alpha - \beta v|) = \frac{t}{mR}$$

Substituting α and β for their value and rearranging:

$$t = \frac{mR}{B^2L^2}\ln\left(\frac{FR}{FR - B^2L^2v}\right)$$

Now plugging in all the values:

$$t = \frac{(0.12)(80)}{(1.5^2)(0.2^2)}\ln\left(\frac{(1.9)(80)}{(1.9)(80) - (1.5^2)(0.2^2)(25)}\right) = \boxed{1.59\text{ s}}$$

Problem 2.
University Physics, Problem 65. Exam Problem

The long, straight wire shown in the figure carries constant current I. A metal bar with length L is moving at constant velocity as shown in the gure. Point a is a distance d from the wire.

a. Calculate the emf induced in the bar.

b. Which point, a or b, is at higher potential?

c. If the bar is replaced by a rectangular wire loop of resistance, what is the magnitude of the current induced in the loop?

Part (a)

When this problem was included on the exam, students were required to use the law
$d\mathcal{E} = \vec{\mathbf{v}} \times \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$
$\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is directed upwards and $d\vec{\mathbf{l}}$ is directed downwards, so $d\mathcal{E} = -vBdl$

$$B = \frac{\mu_0 I}{2\pi r} \implies d\mathcal{E} = \frac{-\mu_0 I v dr}{2\pi r}$$
$$\mathcal{E} = \int_d^{d+L} \frac{-\mu_0 I v}{2\pi r} dr = \boxed{\frac{-\mu_0 I v}{2\pi} \ln|\tfrac{d+L}{d}|}$$

Part (b)

Using the right hand rule, the force on a point charge along the rod would be directed upwards, i.e., in case of a closed circuit, the current would flow upwards. This might confuse some people into saying that point b is at higher potential, but this is incorrect. It is true that current flows from high to low potential but that only applies in circuit. This bar acts as a source of (incuded) emf, which means it's just like a bettery, so current flows from low potential to high potential, i.e. a is at higher potential. Note that most people answered this question with a on the exam because they though that it's at a higher potential because it's closer to the wire, which is incorrect. They got the full grade as the exam did not require a reason, but if it did, they would have gotten a zero. This is a very important point and was worth the clarification.

Part (c)

The $\mathcal{E}$ would be zero. The magnetic field does not change, neither do the area nor the angle. The magnetic field is non-uniform, but it only changes as we move away from the wire not along it. It's easy to prove that the emf is zero by finding the flux through the loop, which will turn out to be constant.