1 Introduction to Dimensional Analysis

1.1 What is dimensional analysis?

Dimensional analysis is a "technique" used in STEM to identify the relationship between different physical quantities, and to verify the correctness (dimensional consistency) of complex equations. Dimensional analysis is very important for any engineering student, and is extremely helpful in this course.

1.2 Why do we need dimensional analysis?

Suppose you're solving a problem and you end up with an expression that looks like this:

$$v_0 = \frac{\mu_0 Q_0^2}{(4\pi\lambda R_{eq}Cd)(R_1+R_2+R_3)} \cdot \frac{d\Phi_B}{dt} \cdot (\oint_A \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}})^{-1} \cdot e^{-t/R_{eq}C}$$

This expression for the velocity might look extremely complicated, and it's unlikely that you will deal with anything like this in this course. The first half of this expression is actually the answer to a problem in the book.
What's the quickest way to verify that this expression is correct? You can go through all your steps (which you should do), but the quickest way is through dimensional analysis. However, dimensional consistency does NOT mean that your solution is correct, it just means that you're on the right track.

1.3 Verifying dimensional consistency

1.3.1 Transcendental functions

The arguments of transcendental functions have to be *dimensionless.* This is the easiest way to catch dimensional inconsistencies.
Transcendental functions include:

- Exponential functions and their inverses (i.e. logarithms). e.g., $e^x, 2^x, \ln(r_a/r_b), \log_2(C_1/C_2), \ldots$
- Trigonometric functions and their inverses. e.g., $\sin(\omega t), \sin^{-1}\theta, \ldots$

Example:

$$q(t) = Q_0 e^{-t/RC} + VC(1 - e^{-t/RC})$$

This is the general solution to a differential equation in chapters 6 and 10.

$$C \cdot [e^{s/\Omega \cdot F}] + V \cdot F([1] - [e^{s/\Omega \cdot F}]) = C \cdot [e^{s/((V/A)\cdot(As/V)}] + V \cdot C/V \cdot ([1] - [e^{s/((V/A)\cdot(As/V)}]) = C + C = [Q]$$

$\therefore$ the equation is dimensionally consistent.

1.3.2 Addition

When adding several quantities, their units *must* be the same.

$$\mathcal{E} = \frac{Q}{C} + L\frac{dI}{dt} + IR$$

This equation will come up again in chapters 10 and 11.

$$\frac{C}{F} + H\frac{A}{s} + A \cdot \Omega = \frac{C}{C/V} + \frac{V \cdot s}{A} \cdot \frac{A}{s} + \frac{V}{\Omega} \cdot \Omega = V + V + V = [\mathcal{E}]$$

$\therefore$ the equation is dimensionally consistent.

1.3.3 Differentiation and integration

When differentiating, just treat it as a ratio. When integrating, don't forget about the unit of the differential (same unit as the physical quantity it represents).
Example:

$$\oint_l \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$$

This is Faraday's law in integral form, and will come up often.

$$\mathrm{V/m \cdot m = V}$$

$$\frac{\mathrm{Wb}}{\mathrm{s}} = \frac{\mathrm{V \cdot s}}{\mathrm{s}} = \mathrm{V}$$

$\therefore$ the equation is dimensionally consistent.

1.4 A complicated example

$$\frac{(T \cdot (m/A)(C^2)}{([4\pi](kg/m)\Omega \cdot F \cdot m)(\Omega + \Omega + \Omega)} \cdot \frac{V \cdot s}{s} \cdot (A/m^2 \cdot m^2)^{-1} \cdot [e^{s/\Omega \cdot F}]$$

$$= \frac{(Wb/m^2) \cdot (m/A)(C^2)}{(kg)(V/A) \cdot (C/V))(\Omega)} \cdot \frac{V}{A} \cdot [e^{s/((V/A) \cdot (A.s/V)}]$$

$$= \frac{(V \cdot s/m)(C)}{(kg))(\Omega)} \cdot \Omega = \frac{(J/C) \cdot (s/m)(C)}{kg} = \frac{(kg \cdot m^2/s^2) \cdot (s/m)}{kg} = m/s = [v]$$

$\therefore$ the equation is dimensionally consistent.

2 SI Units, Prefixes and Important Constants

2.1 Fundamental SI Units

Name	**Symbol**	**Name of Unit**	**Unit Symbol**
Length	l	meter	m
Mass	m	kilogram	kg
Time	t	second	s
Electric current	I	ampere	A

2.2 Derived SI Units

Name	**Symbol**	**Name of Unit**	**Unit Symbol**	**Equivalent Units**
Area	A	squre meter	m^2	
Volume	V	cubic meter	m^3	
Frequency	f	Hertz	Hz	s^{-1}
Mass density	ρ	kilogram per cubic meter	$\mathrm{kg/m}^3$	
Speed, velocity	v	meter per second	m/s	
Angular velocity	ω	radians per second	rad/s	
Acceleration	a	meter per second squared	$\mathrm{m/s}^2$	
Force	F	newton	N	$\mathrm{kg} \cdot \mathrm{m/s}^2$
Work	W	joule	J	$\mathrm{N} \cdot \mathrm{m}$, $\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$
Power	P	watt	W	J/s
Electric charge	Q	coulomb	C	A· s
Volume charge density	ρ_q	coulomb per cubic meter	$\mathrm{C/m}^3$	
Surface charge density	σ_q	coulomb per square meter	$\mathrm{C/m}^2$	
Linear charge density	λ_q	coulomb per meter	C/m	
Potential difference	V	volt	V	J/C
Electromotive force	$\mathcal{E}$	volt	V	J/C
Electric field strength	E	volt per meter	V/m	N/C
Electric resistance	R	ohm	Ω	V/A
Capacitance	C	farad	F	C/V
Energy desnity	u	joule per cubic meter	$\mathrm{J/m}^3$	
Magnetic flux	Φ_m	weber	Wb	$\mathrm{T} \cdot \mathrm{m}^2$, $\mathrm{V} \cdot \mathrm{s}$
Inductance	L,M	henry	H	$\mathrm{V} \cdot \mathrm{s}$ /A
Magnetic flux density	B	tesla	T	$\mathrm{Wb/m}^2$
Wave number	k	1 per meter	m^{-1}	
Radiation Intensity	I	watt per square meter	$\mathrm{W/m}^2$	

2.3 Important Constants

Name	**Symbol**	**Value**
Speed of light in vacuum	c	3×10^8 m/s
Magnitude of charge of electron	e	1.6×10^8 C
Mass of electron	m_e	9.1×10^{31} kg
Coulomb (electric) constant	$K_e = \frac{1}{4\pi\epsilon_0}$	$9 \times 10^{\ 9}$ N $\cdot$ m^2 / C^2
Magnetic constant	$K_m = \frac{\mu_0}{4\pi}$	$1 \times 10^{\ -7}$ T $\cdot$ m / A
Permittivity of free space	ϵ_0	8.85×10^{-12} C^2/N$\cdot$m^2
Permeability of free space	μ_0	$4\pi\times 10^{-7}$ T$\cdot$m/A

2.4 Prefixes

Power of ten	**Prefix**	**Abbreviation**
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	μ
10^{-3}	milli-	m
10^{-2}	centi-	c
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G